Linear optimisation models

- mathematical programming
  - linear objective function
  - linear constraints
- early availability of reliable solver (Simplex, 1947)
- often: efficient allocation of limited resources
  - e.g. satisfying electricity demand at minimum total system cost under constraints (emissions, potentials of technologies, annual capacity additions, financing, ...)

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Linear optimisation models: general form

- objective function:

$$\max or \min f(x_j) = \sum_j c_j x_j \qquad j = 1,\ldots,n$$

- constraints:

$$\sum_j a_{ij} x_j \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b_i \qquad i = 1,\ldots,m$$

- sign restrictions, e.g.: $x_j \geq 0$

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Linear optimisation models: slack variables

- transforming weak inequalities into equations:

  $\sum_j a_j x_j \leq b$ becomes $\sum_j a_j x_j + s = b$ with $s \geq 0$

- result: standard form of an LP

  => ready for Simplex algorithm

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Linear optimisation models: assumptions

- Proportionality (to activity level)
  and additivity (of inputs and outputs)
  - some non-linear variable interaction?
    -> non-linear programming
- Divisibility (no integer values)
  - (some) discrete variables?
    -> integer programming
    -> mixed integer programming

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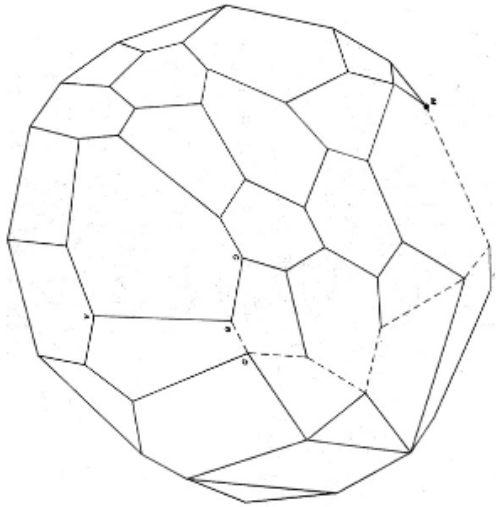
Linear optimisation models: solution geometry

- n-dimensional feasible region
- optimized linear objective
  - solution is never in the interior of the feasible region
  - solution is usually at a corner, but is not generally unique
  - richness of information (many constraints) to avoid extreme solutions and extreme switching behaviour
  - for large scale models, it is often difficult to identify the binding constraints and their shadow prices
  - large scale models may have very long solution times, which often restricts the number of scenarios



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LP models: environmental issues

- environmental constraints and modeling, e.g.
  - emission coefficients
  - transfer coefficients
  - changes in activity levels
- explicit modeling of technological options
  - e.g. merit order of technologies or abatement measures
  - additional bounds (merit order doesn't tell the whole story)
  - cost-effective solutions

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Linear optimisation models: pros & cons

- simplicity (?)
- robust solvers for large-scale applications
  => (local) linearisation can make problems solvable
  -> allows for a lot of detail

- no non-linear functions
  (e.g. convex indifference curves)
- tendency to switching and extreme solutions
- evaluation of costs rather than welfare

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